

UNIT-1

-Antenna And Wave Propagation

A) Antenna Basics

Definition:- It is usually a metallic device for radiating (or) receiving radio waves.

[OR]

An Antenna may be defined as the structure associated with the region of transmission between a guided wave and free spaces.

Antenna Basic parameters:-

- * Frequency
- * wavelength
- * Impedance matching
- * VSWR and reflected power
- * Bandwidth
- * Radiation Intensity

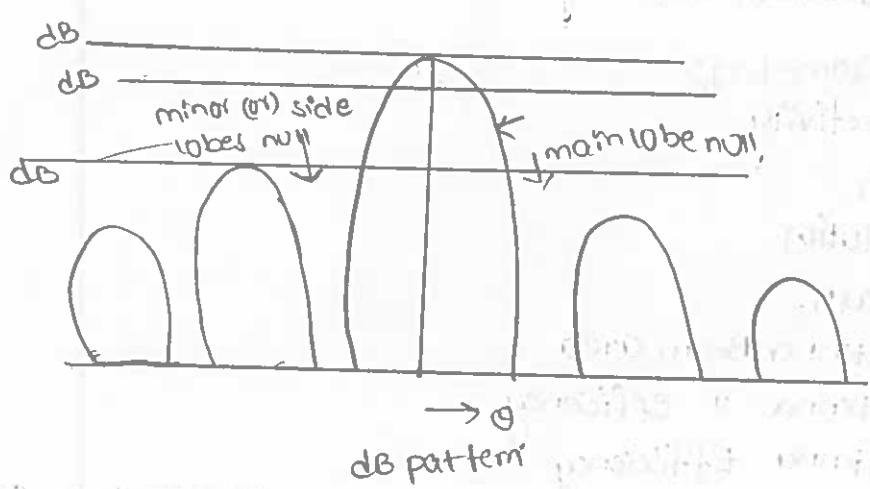
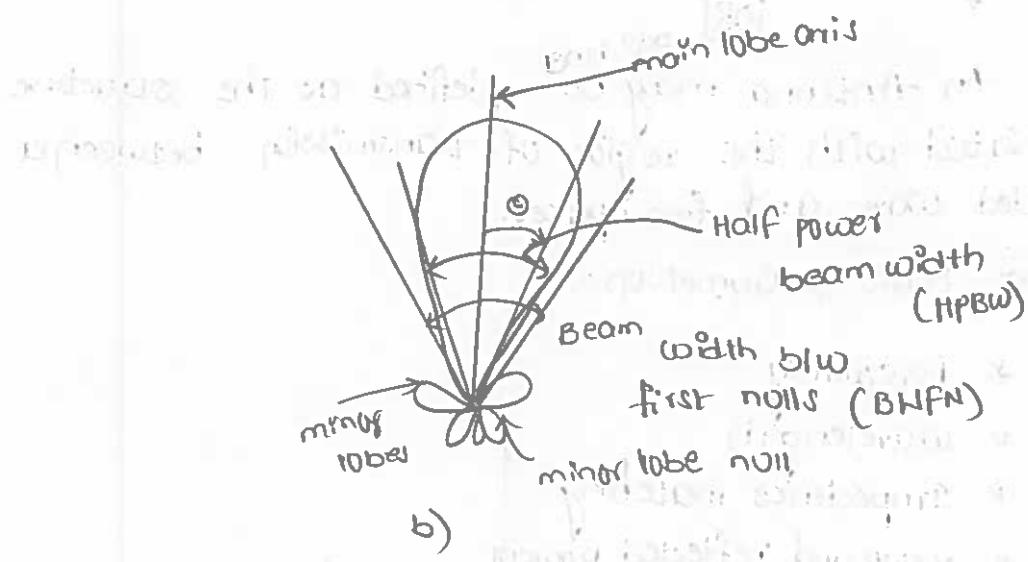
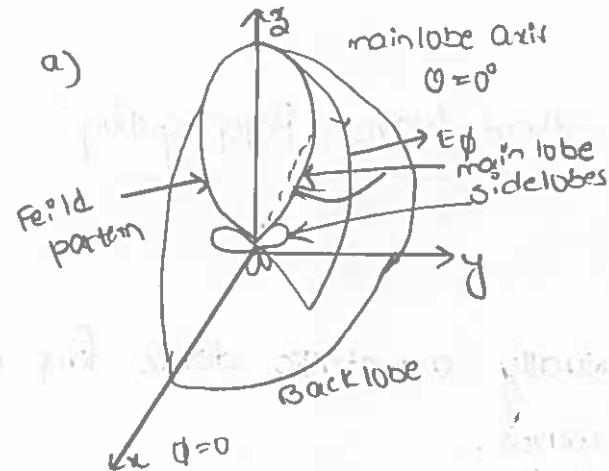
Antenna parameters:-

- * Directivity
- * Gain
- * Resolution
- * Pattern
- * Antenna Beam Area
- * Antenna " efficiency
- * Antenna Efficiency

Now, Let us discuss about antenna parameters in detail.

a) pattern:- It defines as how the field varies from point to point on a spherical surface.

The pattern has its main lobe maximum to



In, 3 direction ($\theta=0$) with minor lobes (side and back) in other directions.

In this pattern it is mainly based on the radiation of the electromagnetic waves, which is called "radiation pattern".

Radiation pattern lobes:-

Different parts of radiation patterns are referred as "lobes".

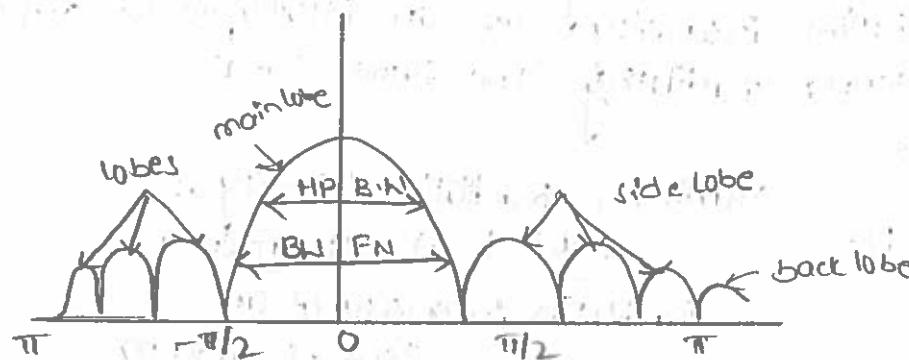


Fig:-1) Linear plot of power pattern

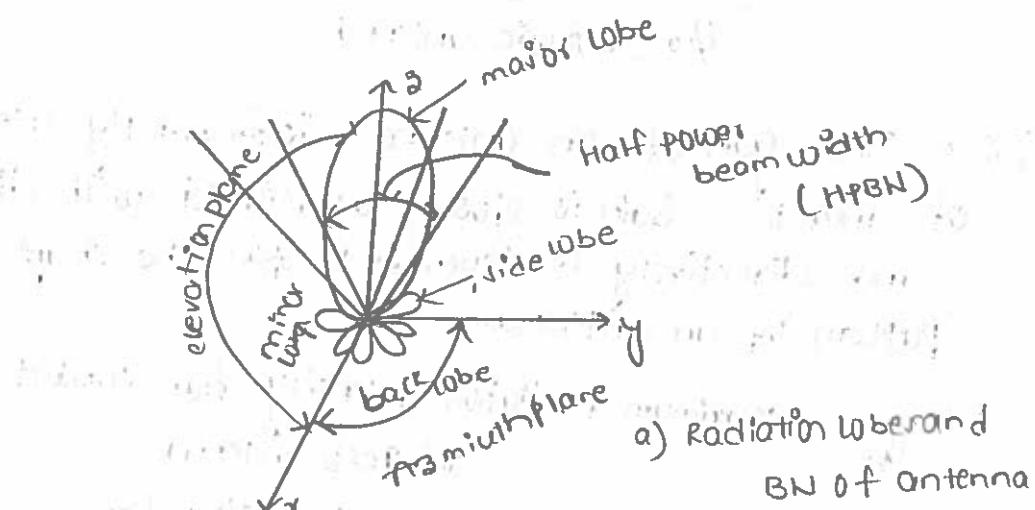


Fig:-a) Radiation lobes and BN of Antenna

* A radiation lobe is a portion of the radiation pattern bounded by regions of relatively weak radiation intensity. The radiation pattern lobes are sub classified into 4 types, such as major lobe, minor lobe, side lobe and back lobe.

* Major lobe is also called as "main beam" and is defined as "the radiation lobe containing the direction of maximum radiation".

- * Side lobe is the adjacent to the main lobe. side lobe occupies the hemisphere in direction of main lobe.
- * Back lobe is mainly refers to a minor lobe that occupies the hemisphere in a direction opposite to that of major lobe.

Directivity (D): It is defined as the ratio of maximum radiation intensity of the subject antenna to the radiation intensity of an isotropic (bi) reference antenna radiating the same total.

i.e,

$$D = \frac{\text{Maximum Radiation intensity of subject (bi) test antenna}}{\text{Radiation intensity of an isotropic antenna}}$$

$$D = \frac{\Phi(\theta, \phi)_{\text{max}} (\text{test antenna})}{\Phi_0 (\text{Isotropic antenna})}$$

Gain:- The gain of an antenna is frequently used as figure of merit. Gain is closely associated with directivity and directivity is dependent upon the shape of radiation pattern of an antenna.

$$G = \frac{\text{maximum radiation intensity from subject (bi) Test antenna}}{\text{maximum radiation intensity from reference antenna with same power input.}}$$

symbolically,

$$G_D = \frac{\Phi_m}{\Phi_0}$$

where; Φ_m is maximum radiation from test antenna

Φ_0 is radiation intensity from lossless isotropic antenna.

Resolution: It is defined as equal to half the beam width between the first nulls. (BNFN/2). Half the beam width between first nulls is approximately equal to the half power beam width (HPBW) (σ_1)

$\frac{BWFN}{3} \approx HPBW$

the product of BWFNLs in the two principle planes of the antenna beam area.

-Antenna Beam Area:- An area $d\sigma$ of the spherical ($0r$) surface of a sphere as seen from the center of the sphere subtends a solid angle ($d\Omega$) from the above figure.

The total solid angle subtended by the sphere is 4π .

Antenna Beam Efficiency:-

It is the parameter that is frequently used to judge the quality of transmitting and receiving antenna.

The beam efficiency is defined as;

BE = Power transmitted within an angle (θ_1)

Power transmitted by the antenna

Antenna Efficiency (η):- It is defined as the ratio of power radiated to the total input power supplied to antenna.

Radiation Intensity:- It is a quantity, which does not depend upon the distance from the radiator and it is denoted by ϕ . It is defined as the "power per unit solid angle".

Effective High (λ) lengths- the term effective length of antenna represents the effectiveness of an antenna as radiator (or) collector of electromagnetic wave energy.

The effective length is nothing but the ratio of induced voltage at the terminal of the receiving antenna under open circuited condition to the incident electric field intensity.

$$\text{Effective length } (L_e) = \frac{\text{open circuited voltage}}{\text{Incident field strength.}}$$

Aperature Antenna:- It is defined as the ratio of power received at the antenna load terminal to the Poynting vector in watts/m² of the incident wave.

thus, Effective Aperature = $\frac{\text{Power Received}}{\text{Poynting vector of incident wave}}$

$$\Rightarrow A_e = \frac{W}{P}$$

$$\Rightarrow W = A_e \cdot P$$

Fields from oscillating dipole:-

Although, a charge moving with uniform velocity along straight conductor does not radiate a charge moving back and forth in simple harmonic motion along the conductor is subject to acceleration, and radiates.

To illustrate radiation from a dipole antenna let us consider that the dipole of fig (a) has two equal charges of opposite sign oscillating up and down in harmonic motion on the electric field.

At time $t=0$, the charges are at maximum separation and undergo maximum acceleration as they reverse the direction.

At this instant, the current is zero.

At an $1/8$ period later, the charges are moving toward each other and at $1/4$ period they pass at the mid point.

As this happens the field lines detach and new ones of opposite sign are formed. At this time the equivalent current (I) is max and the charge acceleration is zero. As time progresses to a $1/2$ period the fields continue to move out as in figure.

Electric dipole in simple harmonic motion

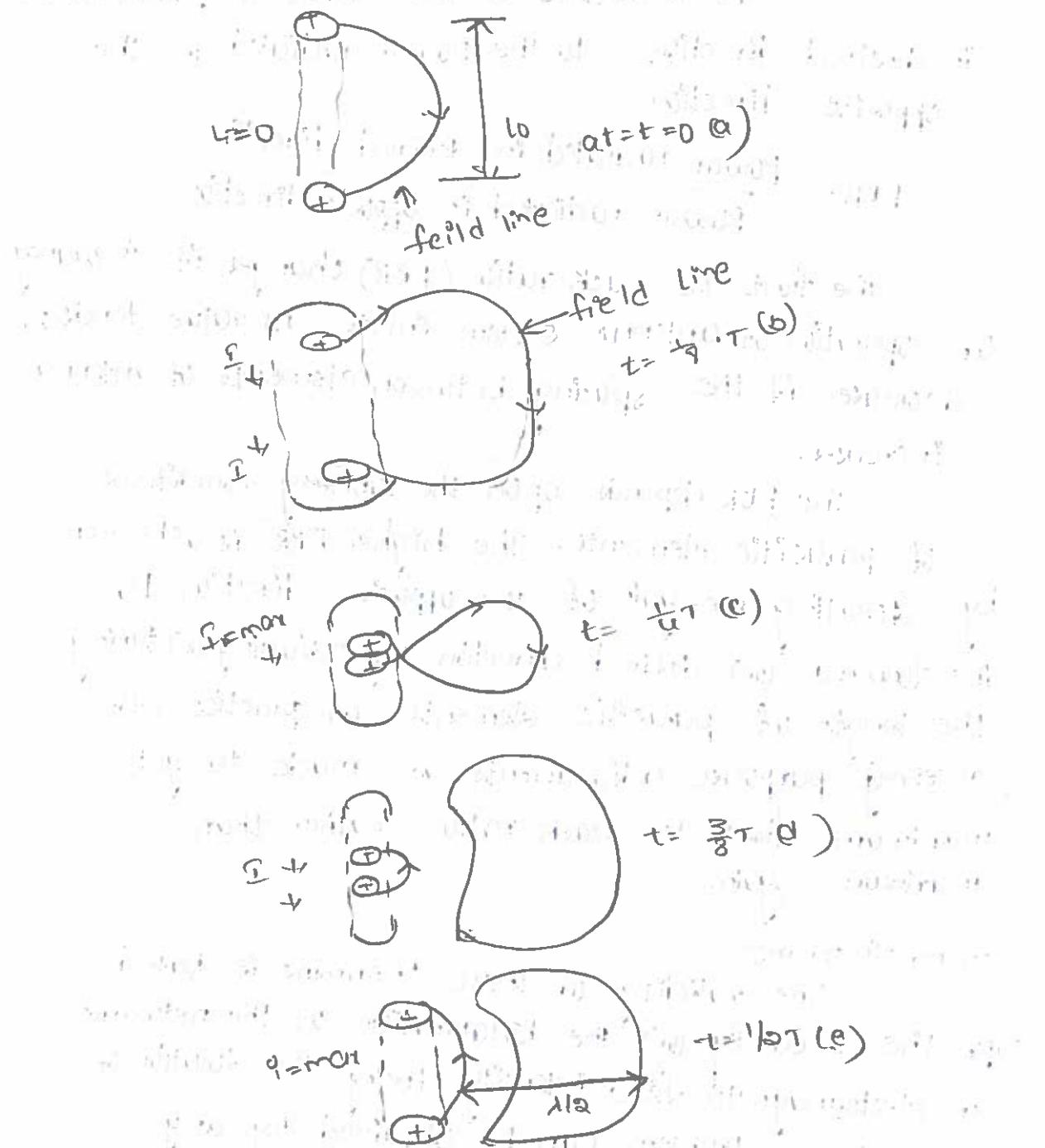


fig:- oscillating electric dipole consisting of two electric charges, in simple harmonic motion, showing propagation of an electric field line and its detachment from the dipole. Arrows next to the dipole indicate current (I) direction.

Front to Back Ratio:-

It is defined as the ratio of power radiated in desired direction to the power radiated in the opposite direction.

$$FBR = \frac{\text{Power radiated in desired direction}}{\text{Power radiated in opposite direction.}}$$

The front to back ratio (FBR) changes, if frequency of operation of antenna system shifts. Its value tends to decrease if the spacing between elements of antenna increases.

The FBR depends upon the tuning conditions of parasitic elements. The higher FBR is achieved by diverting the gain of the opposite direction to the forward (or) desired direction by adjusting (or) tuning the length of parasitic elements. In practice, for received purpose adjustments are made to get maximum front to back ratio, rather than maximum gain.

Antenna theorems:-

The validity of these theorems is based upon the linearity (i) the bilateralism of the networks. In electromagnetic field intensity theory the solution of any antenna problem can be obtained by any antenna problem can be obtained by application of Maxwell's equations and appropriate boundary conditions. The field equations themselves are linear, as long as the constants μ , E and σ of the media involved are truly constant, i.e., do not vary with the magnitude of signal nor with directions, the same theorems can be applied. A few theorems that are most commonly used in antenna problems are as follows:

Superposition theorem:-

In a network of generator and linear impedance, the current flowing at any point is the sum of the currents that would flow if each generator were considered separately, all other generators being replaced at the time by impedance equal to their internal impedances.

Thevenin's theorem:-

If, an impedance ' Z_R ' be connected between any two terminals of a linear network containing one (or) more generators the current, which flows through ' Z_R ' will be the same as it would be if ' Z' were connected to a simple generator, whose generated voltage is the open circuited voltage that appeared at the terminals and whose impedance is the impedance of the network looking back from the terminals will generators replaced by impedances equal to the internal impedance of these generators.

Maximum power Transfer theorem:-

An impedance connected to two terminals of a network will absorb maximum power from the network when, the impedance is equal to the conjugate of the impedance looking back from the output terminals.

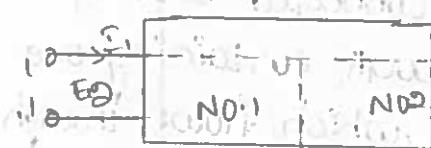
Compensation theorem:- Any impedance in a network may be replaced by a generator of zero internal impedance whose generated voltage at every instant is equal to the instantaneous potential that existed across the impedance, because of the current flowing through it.

Reciprocity theorem:-

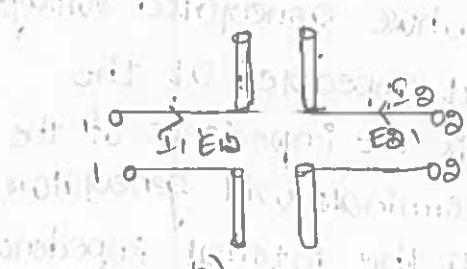
It states, that if an emf is applied to the terminals of an the terminals of another antenna no.s then the equal current both in amplitude and phase will be obtained at the terminals of antenna no.1 if the same emf is applied to the terminals of an antenna no.s.

Assume points:- It is assumed that,

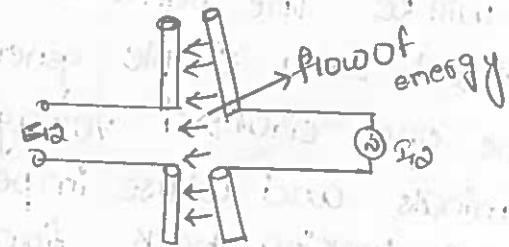
- * emf's are same frequency.
- * media between the two antennas are linear, passive and isotropic.
- * Generators, producing emf and ammeter for measuring the current have zero impedance (r_i) if not then both the generator and the Ammeter impedances are equal.



a)

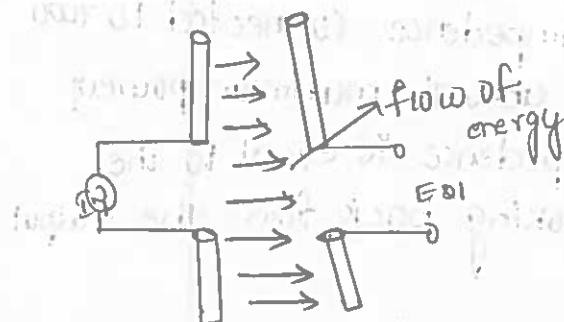


b)



Ant N0.1 medium \rightarrow Ant N0.2

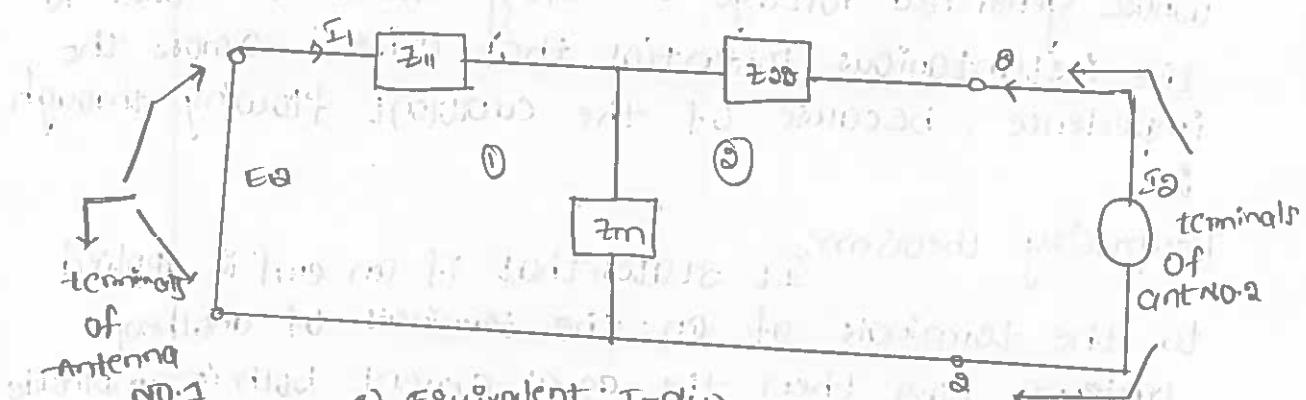
c) If current induced an emf
E02 in antenna N0.2



Ant N0.1 medium \rightarrow Ant N0.2

d) current I_1 inducing

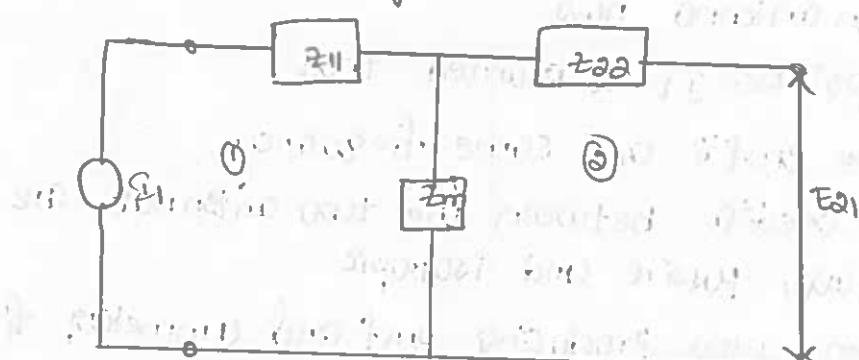
emf E_{01} in antenna N0.2



e) equivalent T-NW

corresponding to 4 terminals

NW figure (c)



f) equivalent T-NW corresponding

to 4 terminals of fig d

Explanation:- From the above figure, let,

- i) a transmitter of frequency 'f' and zero impedance be connected to the terminals of antenna no.2 which is generating a current I_2 and inducing an emf E_{20} at the open terminals of antenna no.1 in figure c.
- ii) Now, the same transmitter is transferred to antenna no.1 which is generating a current I_1 and inducing a voltage E_{10} at the open terminals of the antenna no.2. in fig(d). Thus, according to the statement of reciprocity theorem.

$$I_1 = I_2 \text{ provided } E_{10} = E_{20}$$

Since, ratio of an emf to current is an impedance, therefore, the ratio of $\frac{E_{10}}{I_2}$ is given the name.

Transfer impedance ' Z_{12} ' as in case of Σ and Δ also the ratio $\frac{E_{10}}{I_2}$ as transfer impedance Z_{12} or in II case.

The ratio of voltage (E_1) to the current (I_2) is defined as transfer impedance π (or) Z_{12} .

$$\text{i.e., } \pi = Z_{12} = \frac{E_1}{I_2}$$

Thus, from the reciprocity, it follows that the two ratios i.e., two impedances are equal i.e.,

$$\Rightarrow Z_{12} = Z_{21}$$

$$Z_m = Z_{12} = Z_{21} = \frac{E_{10}}{I_2} = \frac{E_{20}}{I_1}$$

(or)

$$\frac{E_{10}}{I_2} = \frac{E_{20}}{I_1}$$

Proof:- To prove the reciprocity theorem for antenna's,

the space between the antenna no.1 and antenna no.2 are replaced by a network of linear, passive and bilateral impedances shown in figure 'e' and

figure 'f' corresponding to fig 'c' and fig 'd'.

Because, any four terminal network, can be converted into an equivalent T-Net.

$z_{u2} = z_{22}$; self impedance antenna no.1 and
 z_{m12} is mutual impedance b/w two antennas.

I_1 , I_2 is terminals of the antenna no.1

I_2 , I_3 is the terminals of the antenna no.2.

Now, applying Kirchhoff's mesh law to figure e,
from loop (2).

$$\Rightarrow (z_{22} + z_m) I_2 - z_m I_1 = 0$$

$$\Rightarrow I_2 = I_1 \frac{z_m}{(z_{22} + z_m)}$$

from mesh(1)

$$\Rightarrow (z_{11} + z_m) I_1 - z_m I_2 = E_{12}$$

by substituting I_2 's value.

$$\Rightarrow (z_{11} + z_m) I_1 - \frac{z_m I_1}{(z_{22} + z_m)} = E_{12}$$

$$\Rightarrow I_1 \left[\frac{(z_{11} + z_m)(z_{22} + z_m) - z_m^2}{(z_{22} + z_m)} \right] = E_{12}$$

$$\Rightarrow I_1 \left[\frac{z_{11} z_{22} + z_{11} z_m + z_{22} z_m + z_m^2 - z_m^2}{(z_{22} + z_m)} \right] = E_{12}$$

$$\Rightarrow I_1 = \frac{E_{12}(z_{22} + z_m)}{z_{11} z_{22} + z_m(z_{11} + z_{22})}$$

now substituting I_1 's value we get,

$$\Rightarrow I_2 = \frac{E_{12}(z_{22} + z_m)^2 m}{[z_{11} z_{22} + z_m(z_{11} + z_{22})][z_{22} + z_m]}$$

$$= \frac{E_{12} \cdot z_m}{z_{11} z_{22} + z_m(z_{11} + z_{22})}$$

similarly, the current I_3 in the mesh of figure f,
can be obtained (or) by symmetry suffix (2) may be
replaced & and vice versa and from the above eqn,
we can obtain current I_3 .

$$\Rightarrow I_3 = \frac{E_{12} I - z_m}{z_{22} z_{11} + z_m(z_{22} + z_{11})}$$

Thus, early from the ' I_1 ' and ' I_2 ' expressions, they are same except the value of emf (i.e., E_{10} and E_{20}). According to theorem statement, the theorem is proved if we prove,

$$E_{10} = E_{20}, \text{ if } I_1 = I_2$$

applying condition, $I_1 = I_2$

$$\Rightarrow \frac{E_{10} Z_m}{[Z_1 Z_2 + Z_m(Z_1 + Z_2)]} = \frac{E_{20} Z_m}{[Z_1 Z_2 + Z_m(Z_1 + Z_2)]}$$

$$\Rightarrow E_{10} = E_{20}$$

Applications:-

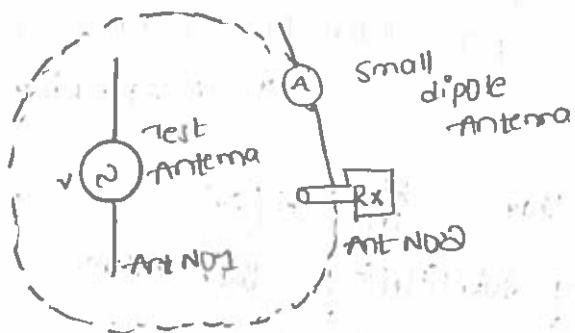
- * Equality of directional patterns.
- * Equality of directivities.
- * Equality of effective lengths
- * Equality of antenna impedance.

Equality of directional patterns:-

The directional patterns of transmitting and receiving antennas are identical if all the media are linear, passive, isotropic and the reciprocity theorem holds good.

Proof:- Under the mentioned conditions it is to be proved that the transmitting and receiving

Fig is.
Directional pattern
measurement for a
transmitting
antenna.



antenna patterns are identical. For this consider figures in which two antennas no. 2 (test antenna) no. 1 (i.e., antenna no. 1 is transmitting and antenna no. 2 is receiving). The pattern may be either field pattern or power pattern, which be either field proportional to square of field pattern. Considering field pattern, keeping the transmitting antenna no. 1 at the center of the observation circle, a receiving antenna no. 2 is moved along the surface of the great observation circle.

Now, if voltage (E) is applied at transmitting antenna no. 1 and the resulting current (I) at the terminals of receiving antenna no. 2 is measured which is the indication of electric field at the location of antenna no. 2. If, the process is reversed i.e., the same voltage (E) is applied to antenna no. 2 and resulting current ' I ' is measured at the test antenna no. 1. This time the receiving pattern of test antenna no. 2 is obtained while previously to that of antenna no. 1.

According to reciprocity theorem of every position of test antenna no. 1 the ratio $\frac{E}{I}$ is same, as was in previous case. Thus, it is proved that radiation pattern of test antenna no. 2 observed by moving receiving antenna no. 2 is identical with the radiation pattern obtained, when the antenna no. 1 is receiving. i.e., when the process is reversed.

Equality of directivities:-

The directivity ' D ' is defined as,

$$D = \frac{\text{Maximum radiation intensity}}{\text{Average radiation intensity}}$$

$$\Rightarrow D = \frac{\psi(0, \phi)_{\text{max}}}{\psi_{\text{av}}} = \frac{\psi_{\text{max}}}{\psi_{\text{av}}}$$

but average;

$$R.I = \frac{\text{Total power radiated, in watts}}{4\pi \text{ in steradian.}}$$

$$\Rightarrow \psi_{\text{av}} = \frac{W}{4\pi} (\text{W/sr})$$

by substituting ' ψ_{av} ' value

$$\Rightarrow D = \frac{\psi(0, \phi)_{\text{max}}}{\frac{W}{4\pi}}$$

$$D = \frac{4\pi \psi(0, \phi)_{\text{max}}}{W}$$

but, the total power radiated is given by radiation intensity $\psi(\theta, \phi)$ integrating over solid angle 4π steradian, i.e.,

$$\Rightarrow W = \iint_{4\pi} \psi(\theta, \phi) d\Omega$$

$$\Rightarrow D = \frac{4\pi \phi(0, \theta)_{\text{max}}}{\iint_{4\pi} \phi(0, \theta) d\Omega} = \frac{4\pi \phi_m}{\iint_{4\pi} \phi d\Omega}$$

$$= \frac{4\pi}{\iint_{4\pi} \left(\frac{\phi}{\phi_m}\right) d\Omega}, \text{ where } d\Omega \text{ is solid angle} = \sin\theta d\theta d\phi$$

$$\Rightarrow D = \frac{4\pi}{\iint \phi_n d\Omega}$$

$$\text{where, } \phi_n = \phi_n(\theta, \phi) = \frac{\phi(0, \theta)}{\phi(0, \theta)_{\text{max}}} = \frac{\phi}{\phi_m}$$

= normalized power pattern

since radiation intensity is a function of θ and ϕ , can be expressed as;

$$\phi = \phi_m f(\theta, \phi)$$

$$\Rightarrow \frac{\phi}{\phi_m} = f(\theta, \phi)$$

$f_n(\theta, \phi) = \text{normalized 3-D power pattern}$

$$\text{As, } \frac{\phi}{\phi_m} = f(\theta, \phi) \text{ then}$$

$$\Rightarrow D = \frac{4\pi}{\iint f(\theta, \phi) d\Omega}$$

Retarded potentials:-

The retarded effect can be accounted by the knowledge that all electric and magnetic effects are propagated at a velocity of c w.r.t the receiver.

The scalar electric potential 'v' and vector magnetic potential 'A' were deal on the basis of the charges being fixed in position 'w' and on the basis of constant charge velocities for 'A'. These potentials were expressed as

$$v = \frac{Q}{4\pi\epsilon_0 R} \text{ volts for concentrated charge}$$

$$v = \frac{1}{4\pi\epsilon_0} \int \frac{P ds}{r} \text{ (volts) for surface charge}$$

$$v = \frac{1}{4\pi\epsilon_0} \int \frac{P v dr}{r} ; \text{ volume charge}$$

and

$$A = \frac{\mu_0 q v}{4\pi r} ; \text{ for a moving concentrated charge}$$

$$A = \frac{\mu_0}{4\pi} \int \frac{IdL}{r} (\text{wb/m}); \text{ Counter line of constant current}$$

$$A = \frac{\mu_0}{4\pi} \int \frac{Jdv}{r} (\text{wb/m}); \text{ volume distribution of current density}$$

The retarded scalar potential can be expressed in terms of retarded time ($t - r/c$). For, volume charge density the expression for retarded scalar potential is given by,

$$V(r,t) = \frac{1}{4\pi\epsilon_0} \int \frac{[P] (t - r/c)}{r} dv$$

where, $V(r,t)$ is scalar electric potential at the point 'p' evaluated at time 't'.

' r/c ' is a retardation time is that time for the effect to be propagated the distance 'r' at velocity 'c'.

The retarded vector magnetic potential can be expressed for volume distribution of current density

$$\text{as } A_p = \frac{\mu_0}{4\pi} \int \frac{[J] (t - r/c)}{r} dv$$

For, a fixed unitary current in a small wire, the difference in distance from points 'p' to various points of given cross-section of wire.

$$A = \frac{\mu}{4\pi} \int \frac{I(t - r/c)}{r} dL$$

for particular case consider current is in 3-3-direction, then

$$\Rightarrow A = \frac{\mu}{4\pi} \int \frac{I_3 (t - r/c)}{r} dL$$

$$I_3 = I_m \sin \omega t$$

then, the modified expression is given as;

$$A = \frac{\mu}{4\pi} \int \frac{I_m \sin \omega (t - r/c)}{r} dL$$

where;

$$[I] = I_m \sin \omega (t - r/c)$$

[I] is retarded current and the bracket is added to indicate that is retarded current.

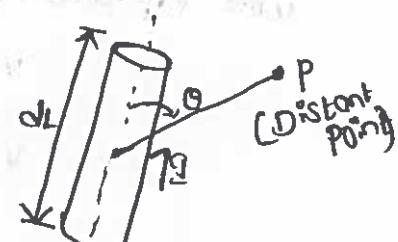


Fig:- 1.8 Current carrying element

For uniform plane wave travelling in \hat{z} direction was given

$$\text{as } E_y = \sin(\omega t - \beta z)$$

Involving the term $\sin(\omega t - \beta z)$ but now the term $\sin(\omega t - \beta r)$ or $\sin \omega(t - \tau/c)$ indicates the travelling of spherical waves in radial direction.

$$\beta = \frac{\omega \pi}{\lambda} = \frac{\omega}{c}$$

$$\sin \omega(t - \frac{r}{c}) = \sin(\omega t - \beta r)$$

by using above condition, for retarded current (I), retarded current density [J] in exponential forms is given as;

$$[I] = I_m e^{j\omega(t-\tau/c)} = I_m e^{j\omega(t-\beta r)} \text{ (Amp)}$$

$$[J] = J_m e^{j\omega(t-\tau/c)} = J_m e^{j\omega(t-\beta r)} \text{ (Amp/m²)}$$

then,

$$[A] = \frac{u}{4\pi} \int \frac{[J]}{r} dr = \frac{u}{4\pi} \int I_m e^{j\omega(t-\tau/c)} dr \text{ (exponential form)}$$

$$(or) [A] = \frac{u}{4\pi} \int \frac{I(t-\tau/c)}{r} dr \text{ (in general)}$$

for sinusoidal current element the retarded potential is

$$= \frac{u}{4\pi} \int \frac{I(t-\tau/c)}{r} ds dl$$

$$= \frac{u}{4\pi} \int \frac{Q(t-\tau/c)}{r} dl = \frac{u}{4\pi} \int \frac{I_m \sin \omega(t-\frac{r}{c})}{r} dl$$

In case of scalar potential

$$[V] = \frac{1}{4\pi \epsilon} \int \frac{[P]}{r} dr = \frac{1}{4\pi \epsilon} \int \frac{P_0 e^{j\omega(t-\tau/c)}}{r} dr$$

where;

$$[P] = P_0 e^{j\omega(t-\frac{r}{c})} = \text{retarded charge density (C/m³)}$$

and two others were followed with some initial and

very gradual and slow

and about the 2nd hour there was a general

followed by another (of 1 hr) in the first 20 min

which took place between 1000 &

$$\frac{d\theta}{dt} = \theta_0$$

(23.30 hrs 10.00)

which followed with a slight increase until 10.

which again [at] 10.00 turned downward, the

rate of change

from 10.00 to 10.30 (10.30 hrs 10.30)

(10.30 hrs 10.30) rate of change at -[1]

and

$$\frac{d\theta}{dt} = \theta_0$$

allowing at 10.30 hrs 10.30

then at 10.30 hrs 10.30

which will now follow below of
the following

$$\frac{d\theta}{dt} = \theta_0$$

$$10 \left(\frac{\theta - \theta_0}{t} \right) \frac{d\theta}{dt} = 10 \left(\frac{\theta - \theta_0}{t} \right) \frac{d\theta}{dt}$$

allowing that θ and t

$$\frac{d\theta}{dt} = \theta_0$$

$$\frac{d\theta}{dt} = \theta_0$$

then becomes $\theta = \theta_0 + \theta_0 t$

(and) which

B) Thin Lineau wire antenna

*Radiation from small electric dipole:-

An alternating current element (α) oscillating electric dipole passes electro magnetic field and let us now proceed to find this fields every where around in free space using the concept of retarded vector potential.

let, the elemental length (dL) of the wire is placed at the origin of spherical coordinate and the current flowing through it.

The length is so short that current is constant along the length. The current element ($I dL$) position is indicated in the fig 1.9.

The aim to calculate the electromagnetic field at point 'p' placed at a distance 'R' from origin.

let, we write the expression for vector potential 'A' at point 'p'. The vector potential 'A' is given as;

$$\Rightarrow A(\gamma) = \frac{\mu}{4\pi} \int J \frac{(t-\gamma/c)}{r} d\gamma$$

As, the vector potential is acting along 'z' direction, it will have only 'z' component. since, the current element is excited by the current $I = I_m \cos \omega t$.

So, replace

$$\Rightarrow \int J(t-\gamma/c) d\gamma = I_m dL \cos \omega(t - \frac{\gamma}{c})$$

$$\Rightarrow A_z = \frac{\mu}{4\pi} \int \frac{I_m dL \cos \omega t'}{r} d(d\gamma)$$

$$A_z = \frac{\mu}{4\pi} \int \frac{I_m dL \cos \omega t'}{r} d(d\gamma)$$

$$A_z = \frac{\mu}{4\pi} \frac{I_m dL \cos \omega t'}{r}$$

$$[\text{since, } A_z = \frac{\mu}{4\pi} \int \frac{J(t-\gamma/c)}{r} d\gamma]$$

$$= \frac{\mu}{4\pi} \int \frac{J(t-\gamma/c)}{r} d\gamma \cdot dL$$

$$= \frac{\mu}{4\pi} \int \frac{I(t-\gamma/c)}{r} dL$$

$$= \frac{u}{4\pi} \int \text{Im} \frac{\sin(\omega t - r/c)}{r} dL$$

$$= \frac{u}{4\pi} \frac{I_0}{r} \sin(\omega t - r/c) dL$$

$$= \frac{u}{4\pi} \cdot \text{Im} \left[\frac{\cos(\omega t - r/c)}{r} dL \right]$$

Now, the magnetic field intensity (H) is obtained from magnetic vector potential $B = \nabla \times A = \mu H$.

Radiated power and Radiated Resistance of

Small electric dipole: Consider a current element placed at a centre of spherical coordinate system.

Then, the power radiated per unit area at point 'P' can be calculated by using Poynting theorem.

The power flow per unit area is given by Poynting vector. The instantaneous power is given by,

$$P = E \times H$$

As, we are considering spherical coordinates (i.e., r, θ, ϕ) for radiated power, then the Poynting vector representation in this coordinates are given as,

$$\Rightarrow P_\theta = -E_\theta H_\phi$$

$$P_\phi = E_\phi H_\theta$$

$$P_r = E_\theta H_\theta$$

In the above mentioned expressions, we use the radial component in term of radiated power.

$$\text{i.e., } P_r = E_\theta H_\theta$$

Substitute value of $E_\theta H_\theta$

$$\Rightarrow \text{Power} = \oint_S P_r ds$$

Substitute up, expression and

$$ds = 2\pi r^2 \sin\theta d\theta$$

$$= \oint_S \left[\frac{\eta_0}{2} \left(\frac{w^2 I^2 \sin^2 \theta}{4\pi c^2} \right)^2 (8\pi r^2 \sin\theta d\theta) \right]$$

$$= \oint_S \frac{\eta_0}{2} \left(\frac{w^2 I^2 d\theta \sin^2 \theta}{16\pi^2 c^2} \right) (8\pi r^2 \sin\theta d\theta)$$

$$= \oint_S \frac{\eta_0 w^2 I^2 d\theta}{16\pi c^2} \sin^3 \theta d\theta$$

$$= \eta_0 \frac{w^2 I^2 d\theta}{16\pi c^2} \oint_S \sin^3 \theta d\theta$$

We know that

$$I_{\text{eff}} = \frac{Im}{f_0}$$

$$Im = \sqrt{\omega} I_{\text{eff}}$$

Therefore,

$$\begin{aligned}\text{Power} &= \frac{\eta_0 \omega^2 (\sqrt{\omega} I_{\text{eff}})^2 dL}{16\pi c^2} \\ &= \frac{\eta_0 \omega^2 I_{\text{eff}}^2 dL}{6\pi c^2} \\ \Rightarrow \text{Power} &= \frac{\eta_0 \omega^2 I_{\text{eff}}^2 dL}{6\pi c^2}\end{aligned}$$

for free space $\eta_0 = 180\pi$ and

$$\frac{\omega}{c} = \frac{2\pi}{\lambda} \text{ j.e., } \frac{\omega^2}{c^2} = \frac{4\pi^2}{\lambda^2}$$

by substituting the value of ' η_0 ' and
 $\frac{\omega^2}{c^2}$! we get;

$$\begin{aligned}\text{Power} &= \frac{180\pi \left(\frac{4\pi^2}{\lambda^2} \right) I_{\text{eff}}^2 dL}{6\pi} \\ &= \frac{80\pi^2 I_{\text{eff}}^2 dL}{\lambda^2} \\ &= 80\pi^2 \left(\frac{dL}{\lambda^2} \right) I_{\text{eff}}^2\end{aligned}$$

$$\text{Power} = 80\pi^2 \left(\frac{dL}{\lambda} \right)^2 I_{\text{eff}}^2$$

Radiation from a half wave dipole ($\lambda/2$ antenna):-

Half wavelength dipole is one of the simplest antenna and is frequently employed as an element of a more complex directional system.

A $\frac{1}{2}$ antenna is also known as "Hertz antenna".
(a) sometimes called as halfwave doublet.
A dipole antenna may be defined as a symmetrical antenna in which the two ends are at equal potential relative to mid point.

Now, we calculate the radiation field of an half wave dipole.

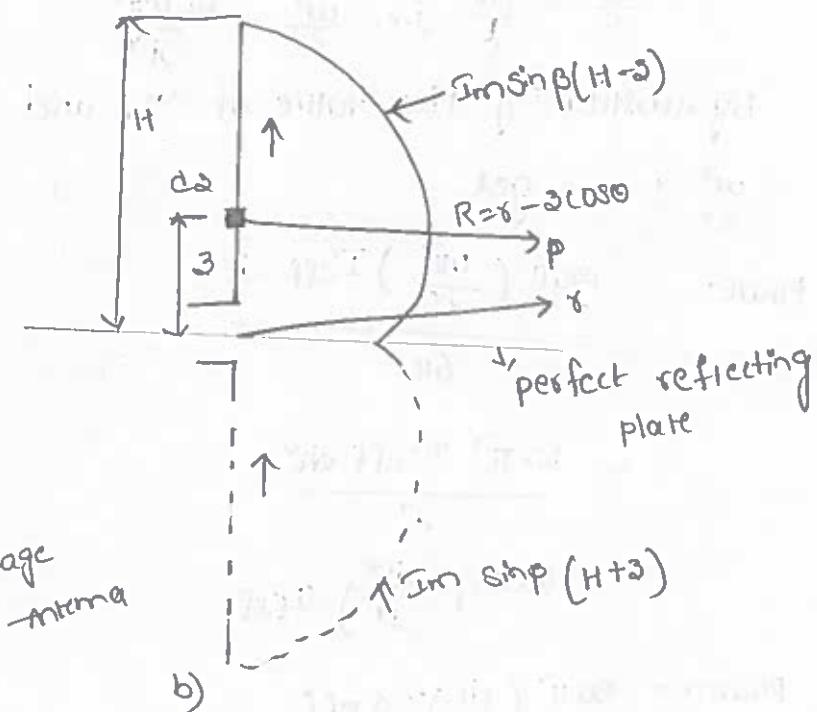
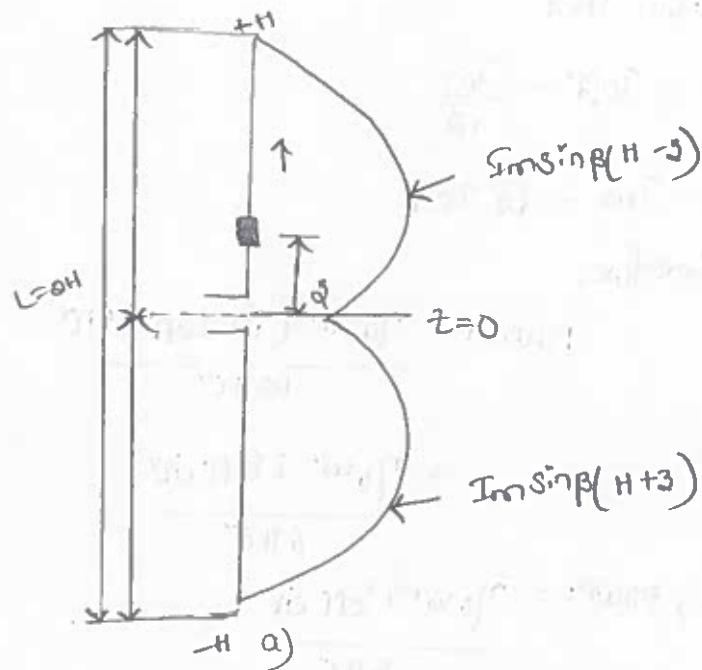


Fig - 1.10 a) Sinusoidal current distribution assumed in centre fed dipole antenna.

b) sinusoidal current distribution assumed in $\lambda/4$ monopole (having only one dipole) antenna.

The dipole usually fed at the centre having maximum current at the centre (i.e., maximum radiation in the plane normal to the axis). The overall specified length is $L=2H$ and the vertical antenna of height $\frac{L}{2}=H$ fed against infinitely large perfectly conducting plane has the same characteristic.

Thus fields originate due to the current element I_{ds} appear to originate from an image 'Id₃' appear

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to originate from an image element situated below the plane, after the reflection from the plane. However, the impedance of vertical feed against the reflecting plane is just half to that dipole of length of $L=sh$.

Since, the current is assumed sinusoidally asymptotically distributed as shown in fig 1.9(a) and fig 1.9(b).

$$I = I_m \sin(\beta(h-z)), \text{ for } z > 0$$

$$I = I_m \sin \beta(h+z), \text{ for } z < 0$$

where, ' I_m ' is current maximum at the current loop.

Now, vector potential at a distance point 'p' due to current element ' Idz ' is given as

$$dA_3 = \frac{\mu I dz e^{-j\beta r}}{4\pi r}$$

Where, 'r' is distance b/w dz to distant point. The total vector potential due to all such current elements at distant point 'p' is given by

$$\begin{aligned} \int dA_3 &= \int_{-h}^h \frac{\mu I dz e^{-j\beta r}}{4\pi r} \\ &= \int_{-h}^0 \frac{\mu I dz e^{-j\beta r}}{4\pi r} dz + \int_0^h \frac{\mu I dz e^{-j\beta r}}{4\pi r} dz \end{aligned}$$

Substitute the value of 'I' according to the range

$$\begin{aligned} &= \frac{\mu}{4\pi} \int_{-h}^0 \frac{I_m \sin(h+z) e^{-j\beta r}}{r} dz + \\ &\quad \frac{\mu}{4\pi} \int_0^h \frac{I_m \sin(h-z) e^{-j\beta r}}{r} dz \end{aligned}$$

$$E_0 = \frac{60 \mu m}{r} \left\{ \frac{\cos(\frac{\pi}{2} \cot \theta)}{\sin \theta} \right\} \frac{V}{m}$$

Since, ' E_0 ' and ' H_0 ' are in the time phase, therefore, the maximum value in time of Poynting vector is

$$P_{max} = |E_0| |H_0|$$

Therefore, the average value in time of Poynting vector is

$$P_{av} = \frac{E_0}{r_0} \cdot \frac{H_0}{r_0} = \frac{1}{2} E_0 H_0 = \frac{P_{max}}{2}$$

$$\Rightarrow P_{av} = \frac{1}{2} E_0 H_0$$

Substitute ' E_0 ' and ' H_0 ' values, then,

$$= \frac{1}{2} \frac{Im}{\pi r^2} \cdot \frac{60\Omega m}{\pi} \left\{ \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta} \right\} \sim$$

$$\Rightarrow P_{av} = \frac{15\Omega m}{\pi r^2} \left\{ \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta} \right\} \sim$$

In practical case current is of importance for measurement purpose. So, $\frac{I}{\sqrt{2}}$ = I_{rms} .

$$\Rightarrow P_{av} = \frac{15(I_{rms})}{\pi r^2} \left\{ \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta} \right\} \sim$$

$$\Rightarrow P_{av} = \frac{30I_{rms}}{\pi r^2} \left[\frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta} \right] \frac{W}{m^2}$$

Fields of short dipole:-

We shall now evaluate the fields everywhere around the short dipole as shown in fig 1.9. It is assumed that the dipole is of length 'l' coincident with the z-axis and its center at the origin. If a current is flowing short dipole the effect of the current is not felt instantaneously at point 'p' but only after interval equal to the time required for the disturbance to propagate over the distance 'r'. This is nothing but retarded effect. Instead of writing the current 'I' in the dipole as $I = I_{meas}$

which implies instantaneous propagation of the effect of the current, the propagation time (τ) retarded time is used as instant.

$$\text{i.e., } [I] = I_m e^{j(\omega t - \beta r)}$$

$$= I_m e^{j(\omega t - \beta r)}$$

where $[I]$ is retarded current w.r.t. that the electric field intensity is given as

$$E = -\nabla V - \frac{\partial A}{\partial t} \text{ V/m}$$

Where;

V = electric scalar potential ϕ ,

A = vector potential at point 'p' in Wb/m

and also magnetic field 'H' at any point 'p' is given as;

$$\nabla \times A = B = \mu_0 H$$

$$\Rightarrow H = \frac{1}{\mu_0} (\nabla \times A) \text{ A/m}$$

Where; $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

using the retarded potential,

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$$E = -\nabla[V] - \frac{\partial}{\partial t} [A] \text{ V/m}$$

$$= -\nabla[V] - \frac{\partial}{\partial t} \omega [A] \text{ and}$$

$$H = \frac{1}{\mu_0} [\nabla \times A] \text{ A/m}$$

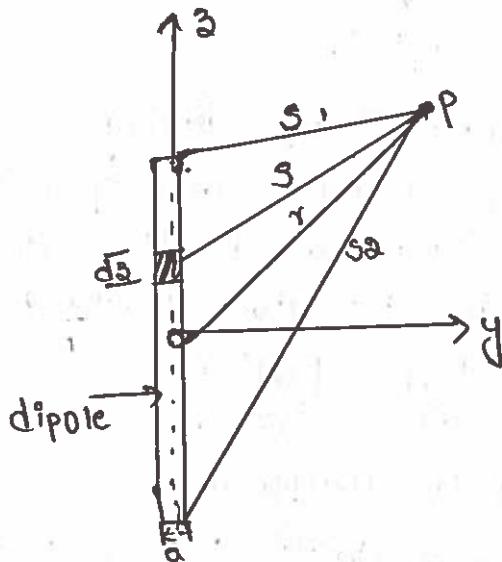


Fig- 1.11(a) short dipole

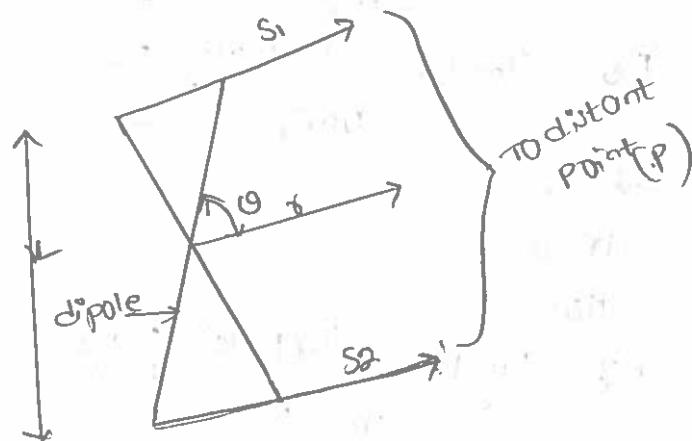


Fig- 1.11(b) short dipole when $\tau \gg 1$

$$\text{Where, } [V] = \frac{1}{4\pi\epsilon_0} \int \frac{[P]}{r} dv \text{ (volts)}$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{p_m e^{i\omega(t-r/c)}}{r} dv$$

$$[A] = \frac{\mu_0}{4\pi} \int \frac{I}{r} dv \text{ (Wb/m)}$$

$$= \frac{\mu_0}{4\pi} \int \frac{J_m e^{i\omega(t-r/c)}}{r} dv \text{ (Wb/m)}$$

NOW, for short dipole located in figure 1.11(a) and (b) the retarded vector potential of the electric current has only one component ' A_3 ' i.e., the current is entirely in z -direction. It follows that retarded vector potential has only ' z ' component given as

$$A_3 = \frac{\mu_0}{4\pi} \int_{-l/2}^{l/2} \frac{[I]}{s} dz$$

where; $[I] = \text{Im} e^{j\omega(t-\tau/c)}$

the retarded scalar potential ψ of a charge distribution is given as;

$$\psi = \frac{1}{4\pi\epsilon_0} \int \frac{[P]}{r} d\tau$$

where; $[P] = p_m e^{j\omega(t-\tau/c)}$

$[P]$ is retarded charge density

since, the region of charge density in the case of the dipole under consideration is confined to the points at the ends, the above equation becomes

$$\psi = \frac{1}{4\pi\epsilon_0} \left\{ \frac{[Q]}{r_1} - \frac{[Q]}{r_2} \right\}$$

summarising all the components,

$$E_r = \frac{\text{Im} [L \cos \theta e^{j\omega(t-\tau/c)}]}{4\pi\epsilon_0} \left[\frac{1}{cr} + \frac{1}{j\omega r^3} \right]$$

$$E_\theta = \frac{\text{Im} [L \sin \theta e^{j\omega(t-\tau/c)}]}{4\pi\epsilon_0} \left[\frac{j\omega}{cr} + \frac{1}{cr^2} + \frac{1}{j\omega r^3} \right]$$

$$E_\phi = 0$$

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = \frac{\text{Im} [L \sin \theta e^{j\omega(t-\tau/c)}]}{4\pi} \left[\frac{j\omega}{cr} + \frac{1}{r^2} \right]$$

case-1:- For field region (or) Radiation zone ($r \gg \lambda$) :-

The field in this region vary as $1/r^2$:

when, r is very large in comparison to λ the higher terms of E' and H' i.e., $\frac{1}{r^3}$ and $\frac{1}{r^4}$ can be neglected in favour of $\frac{1}{r^2}$.

As, such the E_r' component is negelated as can be seen from the summarising all the components which has no $\frac{1}{r^2}$ term.

Therefore, effectively only two components E_θ' and H_ϕ' are contributing the radiation field i.e.,

$$E_\theta' = \frac{\text{Im} [L \sin \theta e^{j\omega(t-\tau/c)}]}{4\pi\epsilon_0} \left(\frac{j\omega}{cr^2} \right)$$

$$\Rightarrow E_\theta' = \frac{jB \text{Im} [L \sin \theta e^{j\omega(t-\tau/c)}]}{4\pi\epsilon_0 cr}$$

$$H_\theta = j\omega \text{Im} \sin\theta e^{j\omega(t-\tau/c)}$$

$$\Rightarrow H_\theta = \frac{j\beta \text{Im} \sin\theta e^{j\omega(t-\tau/c)}}{4\pi r}$$

the ratio of ' E_θ ' and ' H_θ ' is given as

$$\Rightarrow \frac{E_\theta}{H_\theta} = \frac{j\beta \text{Im} \sin\theta e^{j\omega(t-\tau/c)}}{\text{UTR}} \times \frac{4\pi r}{j\beta \text{Im} \sin\theta e^{j\omega(t-\tau/c)}}$$

$$= \frac{1}{E_{DC}} = \frac{36\pi \times 10^9}{3 \times 10^8} = 120\pi \approx 376.8 \Omega$$

$$\Rightarrow \frac{E_\theta}{H_\theta} = 376.8 \Omega$$

This is pure impedance i.e., resistance called intrinsic impedance of the medium.

Case 2:- Near field region $|r| \ll \lambda$: The near field expressions are equations from the summing components (E_r, E_θ and H_θ). For a small value of ' r ', the electric dipole has two components ' E_r ' and ' E_θ ' which are both in phase quadrature with magnetic field as in a resonator. At intermediate distances ' E_r ' and ' E_θ ' may approach time phase quadrature, so, that may total electric field vector rotate in a plane parallel to the direction of propagation, thus showing the phenomenon of cross field.

For, ' E_θ ' and ' H_θ ' components, the near field components are same as the far field patterns as both are proportional to $\sin\theta$. However, near field pattern for ' E_r ' is proportional to $\cos\theta$, as shown in above fig.

Case-3:- Quasi-stationary field region (or) DC case:-

The situation at very low frequencies is referred to as quasi stationary (or) dc case.

As we know that; $[Q] = \frac{[I]}{j\omega}$

$$\Rightarrow [I] = [Q] i\omega = \text{Im } e^{i\omega t},$$

$$\text{Im} = \frac{[Q] i\omega}{e^{i\omega t}}$$

So, that from the summanizing components E_r and E_0 are reduced to

$$E_r = \frac{\text{Im } L \cos \theta e^{i\omega t}}{2\pi \epsilon_0} \left[\frac{1}{C_r} + \frac{1}{j\omega r^2} \right]$$

by substituting Im value

$$= \frac{i\omega [Q] L \cos \theta e^{i\omega t}}{2\pi \epsilon_0 e^{i\omega t}} \left[\frac{1}{C_r} + \frac{1}{j\omega r^2} \right]$$

$$\Rightarrow \frac{[Q] L \cos \theta}{2\pi \epsilon_0} \left[\frac{i\omega}{C_r} + \frac{1}{r^2} \right]$$

$$\Rightarrow E_r = \frac{[Q] \cos \theta}{2\pi \epsilon_0} \left[\frac{i\omega}{C_r} + \frac{1}{r^2} \right]$$

and similarly,

$$\Rightarrow E_0 = \frac{[Q] L \sin \theta}{2\pi \epsilon_0} \left[j\omega \left(\frac{i\omega}{C_r} + \frac{1}{C_r} + \frac{1}{j\omega r^2} \right) \right]$$

$$= \frac{[Q] L \sin \theta}{2\pi \epsilon_0} \left[-\frac{\omega}{C_r} + \frac{j\omega}{C_r} + \frac{1}{r^2} \right]$$

$$\Rightarrow E_0 = \frac{[Q] L \sin \theta}{2\pi \epsilon_0} \left[-\frac{\omega}{C_r} + \frac{j\omega}{C_r} + \frac{1}{r^2} \right]$$

Also, magnetic field is given as;

$$H_\phi = \frac{\text{Im } L \sin \theta e^{i\omega t}}{4\pi} \left[\frac{i\omega}{C_r} + \frac{1}{r^2} \right]$$

$$\Rightarrow H_\phi = \frac{[I] L \sin \theta}{4\pi} \left[\frac{i\omega}{C_r} + \frac{1}{r^2} \right] (\because [I] = \text{Im } e^{i\omega t})$$

At, low frequencies ω , approaches $3\pi^2 0$, therefore, numerators of above equations containing ω terms, may be neglected i.e.,

$$\omega \rightarrow 0, e^{i\omega(t+r)} \rightarrow e^0 = 1$$

So, that

$$[Q] = Q_m e^{i\omega t}, = Q_m \text{ and } [I] = I_m$$

thus, E_r, E_0 and H_ϕ reduces to

$$\Rightarrow E_r = \frac{Q_m L \cos \theta}{2\pi \epsilon_0 r^3}$$

$$E_\theta = \frac{Im L \sin \theta}{4\pi \epsilon_0 r^3}$$

$$H_\theta = \frac{Im L \sin \theta}{4\pi r^3}$$

as long as $r \gg L$

In quasi stationary case, the field decreases as $\frac{1}{r}$ and $\frac{1}{r^3}$, the fields are effectively confined to the vicinity of the dipole and there is negligible radiation.

Fields of thin layer antenna:-

In this section expressions for the far field patterns of thin layer antennas will be developed. It is assumed that the antennas are symmetrically fed at the center by a balanced two wire transmission line. The antennas may be of any length, but it is assumed that the current distribution is sinusoidal. Current distribution measurements indicate that this is a good assumption provided that the antenna is thin i.e., when the diameter is less than say $\frac{1}{100}$.

Thus, the sinusoidal current distribution approximates the natural distribution on thin antennas.

Examples of the approximate natural current distribution on a number of thin linear center fed, antennas of different lengths are illustrated in fig 1.3. The currents are in phase over each $\lambda/2$ section and in opposite phase over the next.

Referring to the fig 1.3, let us now proceed to thin linear centre-fed antenna length L . The z on the antenna referred to a point at a distance r

$$[I] = [i_0 \sin \left[\frac{2\pi}{\lambda} \left(\frac{L}{2} \pm z \right) \right]] e^{j\omega [t - \tau]} e^{j\phi}$$

In above function

$$\sin \left[\frac{2\pi}{\lambda} \left(\frac{L}{2} \pm z \right) \right]$$

is the form of factor for the current on the antenna. The expression $(L/2) \pm z$ is used.

When, $z < 0$ and $(L/2) - z$ is used when $z > 0$. The far fields $E_{\theta 0}$ and $H_{\theta 0}$ at a distance r from the infinitesimal dipole ds are

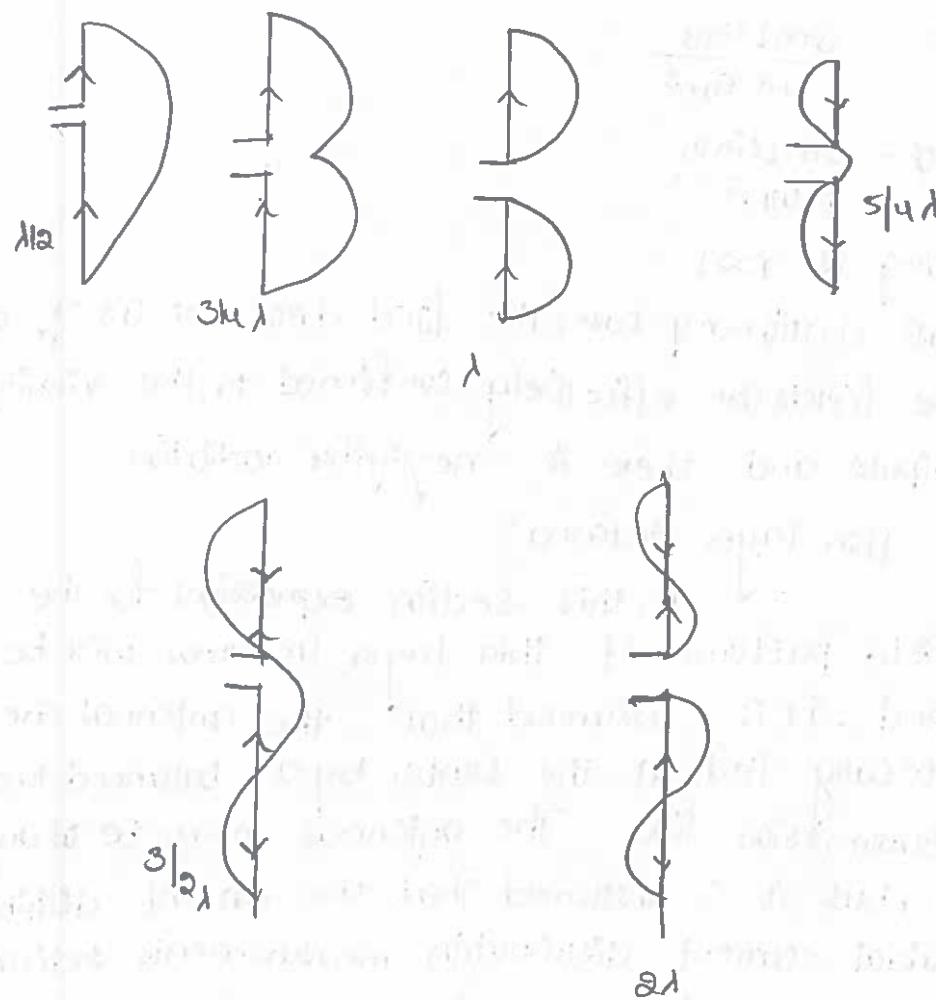


Fig 1.3b:- Approximate natural current distribution for thin linear, center fed antennas of various lengths.

$$dE\theta = \frac{960\pi [I] \sin\theta}{\rho s} ds$$

$$dH\phi = \frac{J[I] \sin\theta}{\rho s} ds$$

$\cos\theta = \frac{1}{2}$ Antenna
when, $L=1/2$, the pattern factor becomes

$$E = \frac{\cos[\frac{\pi}{2} \cos\theta]}{\sin\theta}$$

This pattern is shown in fig 1.5(a). It is only slightly more directional than the pattern of an short dipole which is given by $\sin\theta$.

The beam width between half power points of the $1/2$ antenna is 78° and at half-power points of the $5/16$ antenna is 90° for the short dipole.

Case 2:- full wave Antenna - when, $L = \lambda$

the pattern factor becomes

$$E = \frac{\cos(\pi \cos \theta)}{\sin \theta} + j$$

Case 3:- 3λ/2 antenna

when $L = 3\lambda/2$, the pattern factor is

$$E = \frac{\cos\left(\frac{3}{2}\pi \cos \theta\right)}{\sin \theta}$$

The pattern for this case is shown in figure (c)

with the mid point of the antenna at phase center,
the phase shifts 180° at each null, the relative phase
of the lobes being indicated by the '+' and '-' signs.

Field at any distance from center-fed dipole-

The geometry for the field at the point 'p' from a symmetrical center fed dipole length 'L' with sinusoidal current distribution is presented in figure 1.16. The maximum current is ' I_0 '. It may be shown that the z-component of the electric field at the point 'p' is given by

$$E_z = \frac{-jI_0 \beta}{4\pi} \left[\frac{e^{-j\beta s_1}}{s_1} + \frac{e^{-j\beta s_2}}{s_2} - 2 \cos \frac{\beta L}{2} e^{-j\beta r} \right]$$

The φ component of the magnetic field at the point 'p' is given as;

$$H_\phi = \frac{jI_0}{4\pi r s \sin \theta} \left(e^{-j\beta s_1} + e^{-j\beta s_2} - 2 \cos \frac{\beta L}{2} e^{-j\beta r} \right)$$

If, φ lies on the y-axis ($\theta = 90^\circ$) and the dipole is ' $\lambda/2$ ' long, the E_z becomes

$$E_z = \frac{I_0 \beta}{8\pi r \sqrt{\frac{1}{16} + \gamma_1^2}} \quad \boxed{-360^\circ \sqrt{\frac{1}{16} + \gamma_1^2} - 90^\circ (\text{V/m})}$$

and

$$H_\phi = \frac{I_0}{8\pi r} \quad \boxed{-360^\circ \sqrt{\frac{1}{16} + \gamma_1^2} + 90^\circ (\text{A/m})}$$

where; $\gamma_1 = \gamma/\lambda$

$$Z = 3\pi \Omega$$

I_0 = maximum current = terminal current

At a large distance the ratio of ' E_3 ' as given by ' H_ϕ ' is

$E_3 = Z = 3\pi \Omega$ = intrinsic impedance of space.

The magnitude of ' H_ϕ ' is

$$|H_\phi| = \frac{I_0}{8\pi r} \text{ (A/m)}$$

Equivalence to a loop (short magnetic dipole)

A small loop may be treated as equivalent to a short magnetic dipole. i.e., a small loop of area 'A' having uniform phase current 'I' is replaced by an equivalent magnetic dipole of length 'l' and fictitious magnetic current ' I_m ' as shown in fig 8.53.

Let, we develop a relation b/w the loop and its equivalent magnetic dipole. The magnetic dipole is ' q_m '. • 'l' where,
 q_m is the pole strength and at each end as shown in

figure. The magnetic current is related to this pole strength by

$$I_m = \mu \frac{dq_m}{dt}$$

$$I_m = I_m e^{j\omega t}$$

$$\int_{-T/2}^{T/2} I_m e^{j\omega t} dt = -\mu \int \frac{dq_m}{dt} dt$$

$$\frac{I_m e^{j\omega t}}{(j\omega)} = -\mu q_m$$

$$\frac{I_m}{(j\omega)} = -\mu q_m$$

$$q_m = -\frac{I_m}{j\omega \mu}$$

The magnetic moment of the loop is IA .

Equating this to be the moment of the magnetic dipole.

$$\mu_m l = I \cdot A$$

$$-\left[\frac{Im}{j\omega u}\right] l = I \cdot A$$

(Q8)

$$\begin{aligned} Im l &= -j\omega u I A = -j \cdot 2\pi f \cdot u \cdot I \cdot A \frac{1}{l} = -j 2\pi \cdot \frac{f}{l} \cdot u I A \\ &= -j \frac{2\pi}{l} (8 \times 10^8 \times 10^{-7} \text{ H/m}) I A = -j \frac{2\pi}{l} \cdot (180\pi) I A \\ &= -j 80 \pi \frac{l}{d} I A \end{aligned}$$

$$Im l = -j 80 \pi \frac{l}{d} I A$$

since; $\eta = 180\pi$; $c=f d$ and

$$u = 10^{-7} \text{ H/m}$$

Can be written in retarded form as

$$[Im]l = -j 80 \pi [I] \frac{A}{l}$$

$$\text{where;} \quad [Im] = Im e^{j\omega [t - \frac{r}{c}]}$$

$$\text{and} \quad [I] = I_0 e^{j\omega (t - \frac{r}{c})}$$

Eqs 8.61 & 8.62 are relations between loop area a , and current ' I ' to be its magnetic equivalent magnetic dipole of length ' l ' carrying a function w magnetic current ' Im '.

Radiation Resistance of Loop Antennas:-

In order to calculate the radiation resistance of a loop antenna, the Poynting vector is integrated over a large sphere giving the total power P radiated. This power is then equated to the square of the effective current on the loop times the radiation resistance ' R_r ' i.e,

$$P = I^2 m \nu R_r = \left[\frac{Im}{\sqrt{\epsilon_0}} \right]^2 R_r$$

$$P = \frac{1}{2} I^2 m R_r$$

where;

I_m = peak or maximum current in time on the loop, in A.

R_r = Radiation Resistance, in ohms

P = Radiated power in watts.

The average Poynting vector of a far field is given by;

$$\mathbf{P} = \frac{1}{2} \operatorname{Re}(\mathbf{E} \times \mathbf{H}^*)$$

(a) The radial component are $E_\theta \& H_\phi$ and hence, the radial component of Poynting vector is

$$P_r = \frac{1}{2} \operatorname{Re}(E_\theta H_\phi^*)$$

where, ' E_θ ' and ' H_ϕ^* ' are complex. But $E_\theta = \eta H_\phi$

So, that $P_r = \frac{1}{2} \operatorname{Re}(\eta H_\phi H_\phi^*)$

$$P_r = \frac{1}{2} |H_\phi|^2 \operatorname{Re}\eta$$

(ii)

$$P_r = \frac{1}{2} |H|^2 \operatorname{Re}\eta$$

where;

$H_\phi = H$ = absolute of the magnetic field and

η = Intrinsic impedance of the medium = 120 Ω

for free space putting the absolute value of ' H_ϕ ' from eqn 8.79, we get

$$P_r = \frac{1}{2} \left| \frac{\beta a [i]}{3\pi} J_1(\beta a \sin \theta) \right|^2 \cdot 120\Omega$$
$$= \frac{1}{2} \cdot 120\Omega \cdot \left(\frac{\beta a \sin \theta}{4\pi} \right)^2 J_1^2(\beta a \sin \theta)$$

$$P_r = 15\pi (\beta a \sin \theta)^2 J_1^2(\beta a \sin \theta)$$

The total power radiated ' P ' is obtained by integrating the eqn P_r over a large sphere.

i.e.,

$$P = \int \int P_r d\sigma$$

$$P = \int_0^{2\pi} \int_0^\pi \frac{15\pi (\beta a \sin \theta)^2 J_1^2(\beta a \sin \theta)}{8\pi} r^2 \sin \theta d\theta d\phi$$

Since, $d\sigma = r^2 \sin \theta d\theta d\phi$

$$(ii) P = [V]_0^{2\pi} \int_0^{\pi} 15\pi (\beta a \text{Im})^2 J_1^2 (\beta a \sin \theta) \sin \theta d\theta$$

$$P = 2\pi \cdot 15\pi (\beta a \text{Im})^2 \int_0^{\pi} J_1^2 (\beta a \sin \theta) \sin \theta d\theta$$

$$P = 30\pi^2 (\beta a \text{Im})^2 \int_0^{\pi} J_1^2 (\beta a \sin \theta) \sin \theta d\theta$$

But, a for loop which is small in terms of wavelength,

we may write;

$$J_1(x) \approx \frac{x}{\lambda}$$

$$J_1^2 (\beta a \sin \theta) \approx \left(\frac{\beta a \sin \theta}{\lambda} \right)^2$$

Hence;

$$P = 30\pi^2 (\beta a \text{Im})^2 \int_0^{\pi} \left\{ \frac{\beta a \sin \theta}{\lambda} \right\}^2 \sin \theta d\theta$$

$$= \frac{30\pi^2}{4} (\beta a)^4 (\text{Im})^2 \int_0^{\pi} \sin^2 \theta \sin \theta d\theta$$

$$= \frac{30\pi^2}{4} (\beta a)^4 \text{Im}^2 \frac{2}{3} \int_0^{\pi/2} \sin^3 \theta d\theta = \frac{30\pi^2}{4} (\beta a)^4 \text{Im}^2 \left(\frac{2}{3}\right)$$

$$= 30 \frac{\pi^2}{4} (\beta a)^4 \text{Im}^2 \cdot \frac{4}{3}$$

$$P = 10\pi^2 (\beta a)^4 \text{Im}^2$$

If, there is no loss of power, this power is equal to the power delivered to the terminals of the loop i.e.,

$$\therefore R_t \text{Im}^2 = 10\pi^2 (\beta a)^4 \text{Im}^2$$

$$(iii) R_t = 10\pi^2 (\beta a)^4 = 10\beta^4 (\pi a)^2$$

$$R_t = 10\beta^4 (A)^2 \text{ Ohms}$$

where;

$$A = \pi a^2$$

It can be written in the form as

$$R_t = 80 \times 16\pi^4 \times \frac{A^2}{\lambda^4} = 320\pi^4 \left(\frac{A}{\lambda^2} \right)^2$$

$$(iv) R_t = 80 \left[\frac{2\pi}{\lambda} \right]^4 (A)^2 = 80 \left(\frac{2 \times 3.1416}{\lambda} \right)^4 A^2$$

$$= 80 (6.2832)^4 \left(\frac{A}{\lambda^2} \right)^2 = 80 (1558.56) \left(\frac{A}{\lambda^2} \right)^2 = 81712 \left(\frac{A}{\lambda^2} \right)^2$$

$$(v) R_t \approx \left(\frac{A}{\lambda^2} \right)^2 31200 \text{ Ohms}$$

Alternatively, can also be written as;

$$R_t = 20 \times \left(\frac{8\pi}{1}\right)^4 A^4 = 20 \times 16\pi^4 \cdot \frac{A^4}{1^4}$$

(or)

$$R_t = 320\pi^4 \left(\frac{A}{1^4}\right)^4$$

This is the equation of a small single turn loop antenna circular square with uniform in phase current. If, the loop antenna has 'N' turns wound so that the magnetic field passes through all the loops, the radiation resistance is equal to that of a single turn multiplied by 'N'^2 i.e,

$$R_t = 31,200 \left(\frac{A}{1^4}\right)^4 N^2$$

$$R_t = 31,200 \left(\frac{NA}{1^4}\right)^4 \text{ Ohms}$$

Directivity of loop antennas-

The directivity 'D' of an antenna is defined as the ratio of maximum radiation intensity to the average radiation intensity.

$$\text{i.e., } D = \frac{\text{maximum radiation Intensity}}{\text{Average radiation Intensity}}$$

The maximum radiation intensity for a loop antenna is given by multiplied by γ^2

i.e., $\gamma^2 p_r$ and the average radiation is given by

Eq (a) divided by 4π . i.e. $\left[\frac{P}{4\pi}\right]$. Hence,

$$D = \frac{\gamma^2 \times \frac{15\pi (\text{Baum})^4 J_1^2 (\text{Baum})}{4\pi} \underbrace{\int_0^\pi J_1^2 (\text{Baum}) \sin \theta d\theta}_{\text{max}}}{\frac{30\pi^2 (\text{Baum})^4}{4\pi} \int_0^\pi J_1^2 (\text{Baum}) \sin \theta d\theta}$$

$$= \frac{\{60\pi^2 (\text{Baum})^4 J_1^2 (\text{Baum})\}_{\text{max}}}{30\pi^2 (\text{Baum})^4 \int_0^\pi J_1^2 (\text{Baum}) \sin \theta d\theta}$$

$$D = \frac{2J_1^2 (\text{Baum})_{\text{max}}}{\int_0^\pi J_1^2 (\text{Baum}) d\theta}$$

$$= \frac{2 [J_i^r (\beta a \sin \theta)]_{\max}}{\int_0^{2\pi} J_i^r (\beta a \sin \theta) \sin \theta d\theta}$$

$$D = \frac{2 \beta a [J_i^r (\beta a \sin \theta)]_{\max}}{\int_0^{2\pi} \beta a J_i^r (\beta a \sin \theta) \sin \theta d\theta}$$

$$= \frac{2 C}{\lambda} \left[J_i^r \left(\frac{C}{\lambda} \sin \theta \right) \right]_{\max} \over \int_0^{2\pi} J_i^r \left(\frac{C}{\lambda} \sin \theta \right) \sin \theta d\theta$$

$$D = \frac{2 \frac{C}{\lambda} [J_i^r \left(\frac{C}{\lambda} \sin \theta \right)]_{\max}}{\int_0^{2\pi} J_o(y) dy}$$

This expression is known as Foster's expression for the directivity of a circular loop with uniform in phase current of any circumference ($\frac{C}{\lambda}$). In eqn, angle θ is the value for which the field is maximum.

Case 2:- [for small loop $\frac{C}{\lambda} < \frac{1}{3}$]

The directivity 'D' for a small loop when $\frac{C}{\lambda} < \frac{1}{3}$, the expression for directivity 'D' reduces to

$$D = \frac{2 [J_i^r (\beta a \sin \theta)]_{\max}}{\int_0^{\pi} J_i^r (\beta a \sin \theta) \sin \theta d\theta}$$

Applying the condition for small loop

$$J_i(r) \approx \frac{x}{8}; (r) J_i^r (\beta a \sin \theta) \approx$$

$$D = \frac{2 \left[\frac{(\beta a)^2}{4} \sin^2 \theta \right]_{\max}}{\int_0^{\pi} \frac{(\beta a)^2}{4} \sin^2 \theta \cdot \sin \theta d\theta} = \frac{2 [\sin^2 \theta]_{\max}}{\int_0^{\pi} \sin \theta d\theta}$$

$$= \frac{Q \cdot l}{2 \int_{-\pi/2}^{\pi/2} \sin \theta d\theta} = \frac{Q \cdot l}{2 \cdot \frac{2}{3}} =$$

$$\boxed{D = \frac{3Q}{2l}}$$

This is the same for a small short electric dipole.
This is because, the pattern of short dipole is same
as for a small loop.